Visions of rationality

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The classical view that equates rationality with adherence to the laws of probability theory and logic has driven much research on inference. Recently, an increasing number of researchers have begun to espouse a view of rationality that takes account of organisms' adaptive goals, natural environments, and cognitive constraints. We argue that inference is carried out using boundedly rational heuristics, that is, heuristics that allow organisms to reach their goals under conditions of limited time, information, and computational capacity. These heuristics are ecologically rational in that they exploit aspects of both the physical and social environment in order to make adaptive inferences. We review recent work exploring this multifaceted conception of rationality.

Humans and animals alike make inductive inferences. Firefighters predict how fires will progress from cues such as smoke and roof 'sponginess'[1], while peahens use the elaborateness of peacocks' tails to infer their fitness before deciding whether to mate with them[2]. The cues on which organisms base their inductive inferences are typically uncertain: the old adage aside, sometimes there's no smoke even where there's fire.

How do people make inferences, and are their inferences rational? Most researchers of inference share a vision of rationality whose roots trace back to the Enlightenment. This now classical view holds that the laws of human inference are equivalent to the laws of probability and logic. For French astronomer Pierre Laplace, for example, probability theory embodied human intuition: 'The theory of probability is at bottom nothing more than good sense reduced to a calculus'[3]. Nineteenth-century German philosopher Theodor Lipps wrote that logic 'is nothing if not the physics of thought'[4]. So fundamental was the belief that the mind worked by the rules of probability and logic that when human intuition was observed to deviate from them, the rules themselves were revised[5]. In short, many pre-20th-century thinkers believed that the psychological defines the rational.

Variants of the classical view have flourished in 20th-century psychology. Many researchers maintain the belief that the laws of probability theory and logic at least approximately describe human inference. In the view of Cameron Peterson and Lee Beach, for example: 'Probability theory and statistics can be used as the basis for psychological models that integrate and account for human performance in a wide range of inferential tasks'[6]. According to Jean Piaget, cognitive development culminates in a set of logico-mathematical abilities that essentially reflect the laws of probability and logic. More recently, Lance Rips has argued for the existence of 'mental logic'[7] . Finally, rational-choice theorists and economists often model people's decisions using probability theory as an approximation (e.g. Refs 8,9). Unlike their Enlightenment predecessors, however, these modern researchers see classical models as norms against which human reasoning can be evaluated rather than as codifications of it: when the two diverge, it is concluded that there is something wrong with the reasoning, not with the norms.

In the past 25 years, the idea that human inference can be either defined or described by probability theory and logic has been increasingly challenged. Proponents of the heuristics-and-biases program have argued that inference is systematically biased and error-prone, powered by quick and dirty cognitive heuristics[10]. Numerous departures from classical norms in inductive reasoning – 'cognitive illusions', such as overconfidence, base-rate neglect, and the conjunction fallacy (all discussed in more detail later) – have been attributed to application of these heuristics. A parallel research program has been devoted to accounting for departures of deductive inference from logical norms (for a review of this literature, see Ref. 11).

As the heuristics-and-biases program grew, the view that human reasoning is fundamentally irrational supplanted the belief that it accords with classical rational norms within and outside psychology[12]. In the words of Slovic, Fischhoff, and Lichtenstein: 'It appears that people lack the correct programs for many important judgmental tasks.... it may be argued that we have not had the opportunity to evolve an intellect capable of dealing conceptually with uncertainty'[13]. The conjunction fallacy (see section below on Social Rationality) impelled paleontologist Stephen Jay Gould to speculate: 'Our minds are not built (for whatever reason) to work by the rules of probability'[14]. Some have even argued that deviations from rational norms 'should be considered the rule rather than the exception'[15].

Are violations of rational norms really the rule? Given the analogy between inference and perception behind the illusion metaphor[10], they should be considered an exception. Just as vision researchers construct situations in which the functioning of the visual system leads to incorrect inferences about the world (e.g. about line lengths in the Müller-Lyer illusion), researchers in the heuristics-and-biases program select problems in which reasoning by cognitive heuristics leads to violations of probability theory[12]. However, the

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conclusions they draw from such unrepresentative designs often differ sharply from those drawn by researchers of perception. Vision scientists do not conclude from the robustness of the Müller-Lyer illusion, for instance, that people are generally poor at inferring object lengths. However, many advocates of the heuristics-and-biases program conclude from the cognitive illusions found in laboratory tasks that human judgment is subject to severe and systematic biases that compromise its general functioning[16,17].

How does judgment look when one does not select problems systematically? The use of representative design, which entails simulating real-world conditions of interest in order to test and evaluate human performance[18–20], casts new light on inference. For instance, studies have shown that when people are tested on a representative sample of general knowledge questions, the overconfidence bias found in selected samples of questions disappears (e.g. Refs 21–23; but see also Ref. 24). In a recent meta-analysis of confidence judgments in more than 40 general knowledge tasks, Peter Juslin and colleagues[25] found an average overconfidence of practically zero). In addition, people's estimates of the frequency with which letters of the alphabet appear in various positions within words are better calibrated when participants judge a large, representative sample of letters[26] rather than a small, selected sample[27]. While systematic design, such as that employed in the heuristics-and-biases program, is often desirable when we want to decide between cognitive models, only representative design allows us to evaluate the quality of human judgment in the real world.

Problems with the classical definition of rationality

Despite their disagreements, proponents of the neo-Enlightenment view and the heuristics-and-biases view agree on one critical point: rationality requires reasoning in accordance with the rules of probability theory. Even if we provisionally accept this definition of rationality (which we will challenge shortly), we still see three major problems with it. First, no single conception of probability is shared by all statisticians and philosophers. The applicability of the rules of probability theory to unique events is hotly disputed, with some contending that they apply to unique events and others arguing that they apply only to classes of events[28]. For someone who interprets probability in the latter, strictly frequentistic sense, these rules are irrelevant to the many tasks involving unique events studied in the heuristics-and-biases program. In our view, wherever a norm's applicability depends on our interpretation of probability in this way, we are not justified in treating it as an unequivocal norm of sound reasoning (Refs 29,30; for a recent debate on this point, see Refs 31,32).

The second problem with the classical definition of rationality is its blindness to content and context. In much research on inference, the rules of probability are taken *a priori* as normative, and content is only later filled in. In other words, rather than following the practices of good statisticians, who tailor statistical models to suit specific problems, those who subscribe to this definition of rationality sometimes fail to analyse problem content and people's assumptions about it. Unless this is done, we cannot interpret their judgments. In studies of Bayesian inference, for example, participants might make intelligent assumptions that render some of the given information irrelevant to their judgments, which can be mistaken for neglect of base-rate information[33].

The third and most serious problem we see with the classical definition of rationality is that, beyond the simple problems used in most research, it makes unrealistic demands of the mind. In the real world, matters are more complicated than the simple content-blind norms tested in most laboratory problems assume. Here, Bayes' theorem and subjective expected-utility maximization often become mathematically complex and computationally intractable. Moreover, in many situations, a rational model cannot even be specified because the problem space is unbounded (see Refs 34 Appendix, 35). Expecting people's inferences to conform to classical rational norms in such complex environments requires believing that the human mind is a 'Laplacean demon'[36]: a supercalculator with unlimited time, knowledge, and computational power.

Is there any view between the two extremes we have so far considered, namely, that the mind is an omniscient, omnipotent Laplacean demon or that it simply 'lacks the correct programs'[13] for making many important judgments? Herbert Simon set the stage for what we consider the most promising alternative: 'Human rational behavior… is shaped by a scissors whose two blades are the structure of task environments and the computational capabilities of the actor'[37]. In other words, rationality cannot be defined except by reference to environmental and cognitive constraints. Moreover, rationality is a tool for helping organisms to reach their real-world goals, not necessarily to conform to rational norms. In Simon's own words: 'Reason is… a gun for hire that can be employed in any goals we have'[38]. In the remainder of this review, we describe how recent researchers have used Simon's scissors analogy to fashion a new area of research on human inference.

Bounded rationality

The human mind has to solve important and complex problems – such as deciding whom to marry – under conditions of limited time, knowledge, and computational capacity. Consider Charles Darwin, who methodically listed the pros and cons of marriage and bachelorhood before deciding to marry Emma Wedgewood. Despite his willingness to take such an analytic approach to deciding affairs of the heart, Darwin could not have hoped to make this decision rationally in any classical sense. Suppose that he had attempted to maximize his subjective expected utility. While he deliberated about whether marrying was the right choice, listing each of the infinite conceivable consequences of marrying and not marrying, assigning probabilities to each, and searching for more information about his prospective wives, they would all most likely have married other men (not to mention had children and died). Even if they were infinitely patient, he would still need an infinite number of supercomputers to integrate all of this information for him.

What does Darwin's dilemma illustrate? First, life's important problems cannot necessarily be solved by optimization because the space of possibilities that must be taken into account is often unlimited. Second, even when this space is limited and knowledge is complete, optimization is often impossible to achieve in any real system owing to the computational demands it poses; after all, even Gary Kasparov's arch rival Deep Blue is unable to optimize its moves fully in the well-defined problem space of chess.

Simon proposed that, because of the above constraints, human inference in the real world exhibits 'bounded rationality' rather than the classical rationality assumed by optimal models in psychology, economics, and biology (for a relevant debate, see Refs 35,39). The key feature of bounded rationality is limited information search, which requires some kind of stopping rule. Note that here search can refer either to search for alternatives (e.g. mates), or search for each alternative's values on particular cues (e.g. a potential mate's age, sense of humor, etc.). We now describe various interpretations of bounded rationality, saving the one we favor until last[40].

Some researchers in psychology and economics define bounded rationality as constrained optimization, that is, optimizing relative to a criterion while taking the costs of time, information search, and computation into account (e.g. Refs 34,41–43). This stopping rule – to terminate search when its costs outweigh its benefits – is deceptively simple. In fact, the optimization is simply shifted to the problem of determining when to terminate search, which means that this brand of bounded rationality is saddled with the very intractability that it is intended to eliminate[44]. Unsurprisingly, most economists have not embraced bounded rationality in this form because they 'are in the market for methods for reducing the number of parameters to explain data, and a reduction is not what bounded rationality promises'[42].

The most prominent approach of this type, 'rational analysis'[34,41], is predicated on the assumptions that human cognition is adaptive and that adaptation amounts to optimization. It entails specifying the goals of the cognitive system, developing a formal model of the environment, and deriving the optimal behavioral function based on the goals, formal model, and minimal cognitive constraints. This function is then compared to human performance, and the model duly refined to bring the two into closer correspondence. The cognitive constraints that rational analysis takes into account include deliberation costs and short-term memory limitations. Rational heuristics can conserve cognitive resources by exploiting environmental regularities (e.g. the rarity of most cues and target variables; see Refs 45,46) to simplify the task of optimization.

The rational analysis approach has made impressive progress in developing ecologically appropriate norms to which one can compare human performance in memory, categorization, hypothesis testing and causal inference[34,41,45]. Furthermore, it has demonstrated that some of the most robust findings in cognitive psychology (e.g. power-law learning) can be illuminated by ecological analysis. Still, this approach has important limitations. First, it can be performed only in situations in which an optimal solution can be worked out. Second, devising a computationally tractable rational model requires making vastly simplifying assumptions about the real-world environment (e.g. assuming that cues are independent to trim down Bayesian computations; Refs 34,41). Moreover, because rational analysis starts with a full-blown optimal model made up of mathematical rather than psychological components, it is not well suited to building plausible models of human cognitive processes[35]. As John R. Anderson, who spearheaded rational analysis, has himself observed: 'It is in the spirit of a rational analysis to prescribe what the behavior of a system should be rather than how to compute it'[41].

Some have suggested that the cognitive heuristics identified in the heuristics-and-biases program, such as representativeness and availability, exhibit bounded rationality[10,13,47]. Early research on cognitive heuristics certainly served to demonstrate that human inference does not always conform to classical rational norms. It also encouraged researchers to explore the hypothesis that people rely on cognitive heuristics made up of simple psychological processes rather than formal procedures in order to make inferences. However, to date, the cognitive heuristics posited in this literature have not been formalized such that one could either simulate or analyse mathematically their behavior (for a counter-example, see Ref. 48), thus leaving them free to account for all kinds of performance *post hoc* (for a rebuttal of this point, see Ref. 31). Furthermore, it has not been specified whether or how such heuristics capitalize on environmental structure to make inferences, which is central to Simon's original conception of bounded rationality.

We now describe some models of bounded rationality that capture both the environmental and the cognitive blades of Simon's scissors. Their most critical feature is that they include smart, simple rules for stopping information search. We refer to heuristics based on limited (cue) search as 'fast and frugal'[40].

Fast and frugal heuristics

Which has the larger population: San Diego or San Antonio? If you are not American, you will probably guess San Diego. Why? Because you have heard of it, and the chances are that you have never heard of San Antonio. If you are American, however, you probably recognize both cities, and thus cannot rely on the 'recognition heuristic' to make your choice[40]. In this context, the recognition heuristic can be summarized in one sentence: if you recognize one object and not the other, then infer that the recognized object has the higher value on the target variable; if you do not recognize either object, then guess.

What happens if you recognize both cities? In that case, you have to retrieve information from memory to make an inference. 'Take The Best' is a fast and frugal heuristic for using this information[49]. Imagine that we have a set of objects, all German cities with more than 100,000 inhabitants, and a target variable, population size. Each city can be characterized on a number of binary (or dichtomized) cues, each of which predicts population size to varying degrees. For instance, cities with major-league soccer teams tend to be larger. In Take The Best, the objects are compared on the most valid cue, the second most valid cue, and so on until a cue on which the objects differ is found (see Fig. 1). All that Take The Best needs to learn – or to estimate – is the rank order of cues by validity. Moreover, its stopping rule for information search is very simple: take the best cue (i.e. the most valid one that discriminates) and ignore the rest. The

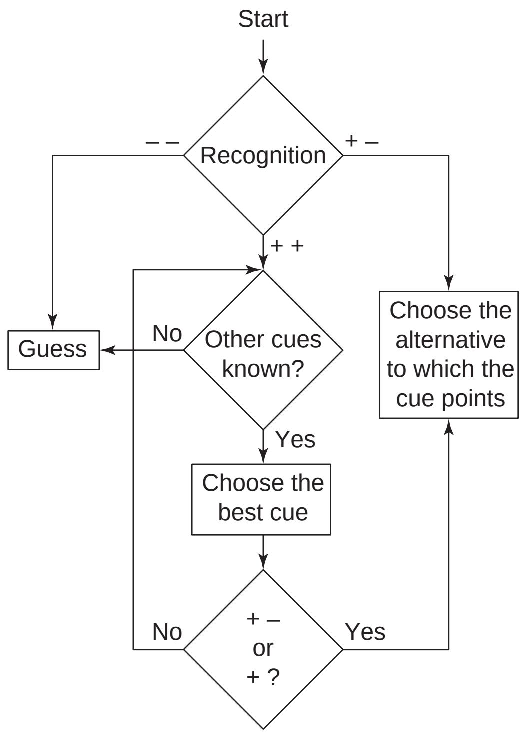

Fig. 1 Flow diagram of a fast and frugal heuristic: Take The Best. (©1996, American Psychological Association; modified, with permission, from Ref. 49.)

first step of Take The Best is always the recognition heuristic, which enables it to exploit ignorance to make smart inferences. (The recognition heuristic could be the first step of other inference strategies as well; see Ref 37.)

Using the recognition heuristic might be smart, but do people actually use it? Empirical results suggest that they do. In one study, American students were asked to make hundreds of inferences about which member of pairs of German cities (e.g. Bielefeld or Munich) was larger. In over 90% of the choices in which recognition discriminated between alternatives, participants in this study opted for the recognized city[40]. Clearly, a person who recognizes all objects cannot use the recognition heuristic (which in the German city environment generally discriminated well between larger and smaller cities, for this American sample). A counter-intuitive implication of this is that someone with less knowledge can actually make more accurate inferences under some conditions, a prediction supported by empirical results (the 'less-is-more effect'; Refs 40,49).

When inferring target variables as diverse as population sizes and high-school drop-out rates, computer simulation results show that Take The Best roughly matches or outperforms in inferential accuracy a number of linear models that integrate across all cues, such as multiple regression and a unit-weighted linear model called 'Dawes' rule'. Even more surprisingly, the 'Minimalist' heuristic, a poor cousin of Take The Best that selects cues in random order, also fares well relative to these computationally more expensive algorithms. Take The Best thrives particularly well compared with integration algorithms when generalizations have to be made (i.e. when test set ≠ training set) rather than when data have to be fitted (i.e. when test set = training set). This is because algorithms that integrate all available information, such as multiple regression, tend to suffer from overfitting, whereas Take The Best relies disproportionately on cues that exhibit greater invariance, at least in the data environments in which it has been tested so far.

Table 1 shows the results of a simulated competition (excluding the recognition heuristic) between Take The Best, Minimalist, Dawes' rule, and multiple regression in which the target variable was rates of homelessness in 50 US cities. The left column indicates the average number of cues that each algorithm had to look up (out of a total possible of six), which was roughly the same in both types of competition. The center and right columns show the percentages of correct binary inferences each algorithm made when the test set was equal to and not equal to the training set, respectively.

Take The Best and Minimalist are clearly more frugal in their use of information than the two integration algorithms, yet are about as accurate as the others. Even more remarkably, Take The Best actually outperforms multiple regression when generalization is required.

Ecological rationality

Fast and frugal heuristics can perform about as well as algorithms that require much more information, and in a serial architecture, more time. What is their secret? The answer lies in their 'ecological rationality'. Such heuristics capitalize on environmental regularities to make smart inferences. For instance, the recognition heuristic exploits the fact that our ignorance is often systematically related to variables that we want to infer (for example, we are more likely to recognize big cities, companies, and universities than small ones).

Mathematical analysis can help us to understand where and why Take The Best can (or cannot) be more accurate than a weighted linear model in which the weights are the

Table 1. Inferring homelessness: a competition between algorithms[a]

Algorithm	Average number of cues looked up	Percent correct binary inferences: Test set the same as training set	Test set different from training set
Minimalist	2.1	61	56
Take The Best	2.4	69	63
Dawes' rule	6	66	58
Multiple regression	6	70	61

[a] *Modified from Ref. 50.*

correlations between cues and the target variable[40,51]. The set of non-redundant binary cues in any environment is finite. Where known cues are abundant (i.e. their number approaches this finite maximum), weighted linear models tend to be more accurate (and their accuracy approaches 100%), whereas where known cues are scarce (i.e. their number is small relative to $\log_2 N$, where N is the total number of objects), Take The Best is on average more accurate. In addition, when the weights are non-compensatory; that is, the weight of each cue exceeds the weights of all those below it in the cue order, the faster and more frugal Take The Best cannot be surpassed by any weighted linear model.

Because the information available in the environment (and in the organism's memory) is often scarce, Take The Best might do well in a wide range of real-world situations. However, no single heuristic can make good decisions in every environment because ecological rationality necessarily implies specificity. The more ecological assumptions are built into a heuristic, the less well it will generalize to environments in which those assumptions are not met. Thus, the mind's 'adaptive toolbox' most likely includes a panoply of heuristics suited for use in different situations[52–54]. As candidate tools, we and our colleagues have developed fast and frugal heuristics for a variety of problems[40], including categorization (see also Ref. 55), mate choice, and quantitative estimation.

In natural environments, information comes in some forms and not others. Ecologically rational heuristics not only take specific environmental structures for granted, but are tuned to work on specific information representations. Unlike probability theory, real systems – whether computers or brains – are not indifferent to how numerical information is represented. For instance, a pocket calculator has an algorithm for multiplication. However, because it is designed to work on numbers entered in base 10 rather than base 2, it would appear to have no algorithm for multiplication at all if one gave it binary numbers as input[56].

To what representations of numerical information might our cognitive algorithms be tuned? The problems typically used in research on inductive inference express information in terms of probabilities or percentages, which are historically a very recent invention. They would not have been encountered in any form in the environments of our evolutionary ancestors, nor can they be directly experienced today, notwithstanding their ubiquity in the media. A more naturalistic way to represent numerical information is in natural frequencies: absolute frequencies that have not been normalized with respect to the base rates (see Box 1). From these considerations we can predict that our cognitive algorithms are more probably designed to reason about numerical information in the form of natural frequencies rather than probabilities.

One of the key findings of the heuristics-and-biases program is that people overweight new data relative to base rates in judging posterior probabilities in Bayesian inference problems (e.g. the probability that a person who tests positive for HIV really has it), which is generally referred to as 'base-rate neglect'[10]. Some of these results may be attributable to the fact that the Bayesian model taken as normative ignores relevant aspects of content or context[33,61]. Still, there are problems in which the Bayesian answer seems appropriate and yet to which people give decidedly non-Bayesian responses. How well would people solve these problems if the information were presented in terms of natural frequencies rather than probabilities? As it turns out, they look much more like Bayesians (see Box 1 and Ref. 58). The reason seems to be computational simplicity: whereas plugging the necessary probabilities into Bayes' theorem requires several steps of multiplication and division, computing posterior probabilities from natural frequencies boils down to simply dividing the number of hits (e.g. people who test positive and really have HIV) by the sum of hits and false alarms (e.g. all people who test positive). In other words, the frequency representation does part of the work for us.

Teaching people to convert probabilities into natural frequencies has been shown to be a powerful tool for training students in Bayesian reasoning[59]. Natural frequency representations also help experts, such as physicians, to make diagnostic inferences[60] and have immediate applications in other contexts, for instance, in helping AIDS counselors and their patients interpret HIV test results[61].

Social rationality

So far we have characterized Simon's ecological blade strictly in terms of the physical environment, but it reflects the social environment (the world of other organisms) as well. We begin our discussion of 'social rationality' by describing some situations in which adhering to social norms is rational although it conflicts with internal consistency, which is often seen as the defining feature of rational choice in decision theory and behavioral economics.

In real-world social contexts, consistency in choice is not always in one's best interest. In competitive situations, it is sometimes desirable to exhibit adaptively unpredictable, or protean, behavior, so that other people and animals cannot predict what one will do[62,63]. For example, our chances of winning a tennis match would be compromised if our opponent knew a stable, consistent order in which we chose to serve to the left or the right during a match. Similarly, when being pursued by a predator that might be able to outrun it, a prey animal would be unwise to flee along a straight, predictable path, even if not doing so means taking longer to cover the same distance[62].

One of the basic principles of internal consistency in choice is known as 'Property α'. Informally speaking, it requires that if you choose A over B, you should do so independently of the other alternatives in the choice set. At first blush, one might believe that all violations of Property α are irrational. However, our social values and goals sometimes conflict with this principle. Imagine, for example, that you are at a dinner party. At dessert, it looks as if there are fewer pastries than there are guests. By the time the dessert tray gets to you, there is only one pastry left, a chocolate eclair. If you know that another of the guests has not yet taken a dessert, out of politeness you might choose to have nothing over having the eclair. However, if the host were to replenish the pastry supply, you might well choose to eat that same eclair over having nothing. In other words, after choosing B (nothing) over A (the eclair), you might choose A over B just because other items were added to the choice set.

Box 1. Anyone can be a Bayesian

Consider the way in which information is represented in the following Bayesian inference problem (adapted from Ref. a):

> The probability of breast cancer is 1% for a woman at age 40 who participates in routine screening. If a woman has breast cancer, the probability is 80% that she will have a positive mammography. If a woman does not have breast cancer, the probability is 9.6% that she will also have a positive mammography.
>
> A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer? (Answer: ____%)

Inserting the given numbers into Bayes' thereom (see Fig.) gives a posterior probability, p(cancer|positive), of 7.8%. However, 95 out of 100 physicians who solved this problem estimated the probability to be between 70% and 80% (Ref. a), an order of magnitude greater than that given by Bayes' theorem. This is a gross error, one that in a real medical context could have serious consequences for patients' well being (at least until a biopsy can be performed). This apparent example of base-rate neglect seems to justify the dire conclusions about human rationality drawn in the heuristics-and-biases program.

However, notice that the mammography problem above is presented in terms of single-event probabilities (e.g. that a particular woman has breast cancer). Human cognitive algorithms for this type of inference, if they exist, are most likely to be designed to operate on numerical information in the form in which humans have gathered it over evolutionary history – natural frequencies; that is, absolute frequencies that have not been normalized with respect to the base rates. Here is how the mammography problem looks when expressed in natural frequencies:

> Ten out of every 1000 women at age 40 who participate in routine screening have breast cancer. Eight out of these ten women with breast cancer will get a positive mammography. Of the 990 women without breast cancer, 95 will also get a positive mammography.
>
> Here is a new representative sample of women at age 40 who got a positive mammography in routine screening. How many of these women do you expect actually to have breast cancer? (Answer: ____ out of ____)

When students who had never heard of Bayesian inference were given problems like this one, they responded like Bayesians in 46% of problems, whereas students who received them in probabilities solved only 16% correctly[b]. Among physicians with an average of 14 years professional experience, the benefit of frequency information was equally strong (again 46% made Bayesian responses compared with 10% given probabilities[c]). In a less complex medical decision-making problem, the percentage of Bayesian solutions found with frequencies rose to 76% (Ref. d).

Why should people reason so much better when given frequencies rather than probabilities? Imagine two people trying to solve the mammography problem (see Fig.). While the person on the left struggles to combine the probabilities according to Bayes' theorem, the person on the right simply has to divide the number of hits (women with cancer and positive mammography) by the total number of hits and false alarms (all women with positive mammography). Thus, instead of taking in frequency information, converting it to probabilities, and plugging them into Bayes' theorem, the mind can simply tally the frequencies and perform a simpler computation. This is an example of how cognitive algorithms make the environment do some of the work for them: natural frequencies carry base-rate information without explicitly representing it.

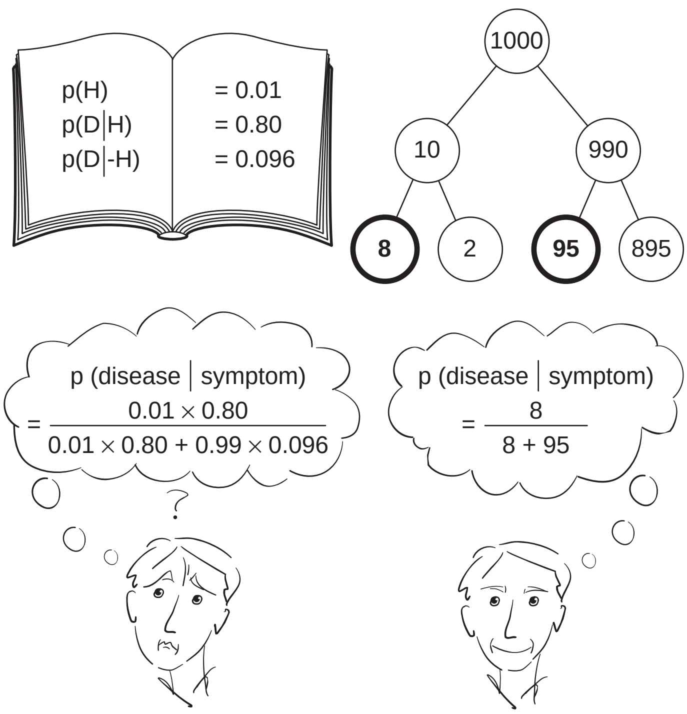

Fig. Differential complexity of Bayesian computations based on probabilities (left) and natural frequencies (right). (see text for explanation.) (©1995, American Psychological Association; modified, with permission, from Ref. b.)

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We have probably all violated Property α in similar situations. Does this make us irrational? Not if we take the social environment into account[64]: being polite pays off. Not being so could anger others and lessen the chances that other people will cooperate with us in the future. Thus, for many of us, violating the social rule 'don't take the last

Outstanding questions

- It has been proposed that the mind has an 'adapative toolbox' of specialized cognitive heuristics suited to different problems[52–54]. What heuristics are in the adaptive toolbox? What environmental cues might determine whether one heuristic is triggered as opposed to another?
- What are the structures of the environments in which specific heuristics perform well or badly, and why? To what extent are people sensitive to these structures and in what terms can we describe them?
- The recognition heuristic relies on recognition memory, which develops early ontogenetically and arose early phylogenetically. What other fundamental psychological abilities might serve as building blocks for fast and frugal heuristics?
- What criteria other than accuracy (e.g. speed, computational complexity) are relevant to evaluating the performance of different heuristics? How well do fast and frugal heuristics fare relative to optimal ones by these criteria? Which heuristics come closest to mimicking human performance on these various measures?
- What role do emotions and culture play in bounded rationality? How might specific emotions, such as love and disgust, help people to make adaptive decisions (e.g. by stopping information search)? Does following social norms provide a fast and frugal way to bypass deliberation by the individual?

piece of cake' carries higher costs than violating Property α, which, after all, will only offend a handful of decision theorists and economists. Of course, we can always account for this example of inconsistency by arguing that the nature of the eclair changed with the change in the choice set; that is, it ceased to be the last dessert. But the point here is that if we allow the meeting of social expectations into our definition of rationality, then we can predict such choices rather than having to explain them *post hoc*.

Another context in which being socially rational requires deviating from a content-blind norm is in the famous 'Linda problem'. In this problem participants read: 'Linda is 31 years old, single, outspoken, and very bright… As a student, she was deeply concerned with issues of discrimination and social justice…' They are then asked to choose which hypothesis is more 'probable': that Linda is a bank teller (T) or that Linda is a bank teller and is active in the feminist movement (T+F). In most studies, 80–90% of participants rank T+F as more probable than T (Ref. 65). This effect – known as the conjunction fallacy – is widely interpreted as a violation of the conjunction rule, according to which the probability of a conjoint event cannot exceed the probability of any of its constituents.

In the Linda problem, participants have to infer what the experimenter means by 'probable,' a term that in natural language has multiple, related meanings, most of which cannot be reduced to mathematical probability (e.g. 'plausible' or 'conceivable'). Which of these meanings do they infer? One possible answer can be derived from Paul Grice's theory[66] of conversational reasoning, which holds that it is reasonable for the audience (participant) to assume that the communicator (experimenter) will follow certain social rules governing communication. If they assume that the 'relevance maxim', by which the audience expects the communicator's contribution to be relevant to the conversation, applies in the Linda problem, participants should infer that 'probable' does not refer to mathematical probability, because a mathematical interpretation would render the description of Linda irrelevant to the requested judgment[67].

Based on this analysis, one can construct a social context in which people following the relevance maxim are more likely to infer a mathematical meaning. If, immediately before the probability judgment, participants are asked for a judgment that renders Linda's description relevant – such as a typicality judgment (i.e. 'How good an example of a bank teller is Linda?') – then adherence to the conjunction rule should increase. Indeed, this is what Hertwig observed[68]: among participants asked to make a typicality judgment first, the percentage following the conjunction rule was on average 40 percentage points higher than that among participants who made the probability judgment immediately. It has also been shown that asking participants to estimate frequencies in conjunction problems (e.g. '*How many* people like Linda are bank tellers?') dramatically increases the percentage of judgments consistent with the conjunction rule[65,69], perhaps because the frequency representation eschews the ambiguity of the term 'probable'[68]. Conversational analysis has revealed still other relevance-preserving inferences drawn in the Linda problem that lead participants to violate the conjunction rule (e.g. Ref. 70).

Such re-analyses of apparent cognitive biases highlight the hazards of confusing with irrationality the human ability to make intelligent semantic and pragmatic inferences[71]. Many researchers are pushing the limits of our knowledge about social rationality in still other ways by studying, for instance, how people reason about deontic conditionals (relating to duty or obligation) and social contracts[53,72–74], and how emotions, traditionally thought to undermine reason, might actually help people to think and decide rationally[75,76]. This research demonstrates that social values and goals deserve a place in both cognitive explanations and definitions of rationality.

Conclusion

We began by challenging the classical vision of human rationality as adherence to the norms of probability theory and logic. Not only are these norms inherently problematic when applied without regard to content and context, but they fail to capture what it means to be rational either in ancestral environments or in the modern world. The mind has evolved to tackle important adaptive problems, not to solve mathematical brain-teasers. We argue that to discover how the mind works, and how well, we need to understand how the mind functions under its own constraints – its bounded rationality – and how it exploits the structure of the social and physical environments in which it must reach its goals – its ecological rationality. By adding these perspectives to our theoretical scope, we broaden and deepen our vision of rationality.

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Stereovision: beyond disparity computations

Barton L. Anderson

One of the most powerful sources of information about three-dimensional (3-D) structure is provided by stereovision (or stereopsis). For over a century, theoretical and empirical investigations into this ability have focused on the role of binocular disparity in generating percepts of 3-D structure. Recent work in image segmentation demonstrates that stereovision can cause large changes in perceptual organization that cannot be understood on the basis of binocular disparity alone. It is argued that these phenomena reveal the need for theoretical tools beyond those that have dominated the study of visual perception over the past three decades.

The power of stereopsis in generating percepts of three-dimensional (3-D) structure is now so familiar that it has become a staple of our entertainment diet. It is difficult not to marvel at the transformation of flat, two-dimensional images into a full 3-D percept that causes objects to 'jump off' the page. This review describes some recent results that reveal the potency of stereovision in shaping our perception of the world, and sketches some general theoretical insights about visual processing that have been gained from this field of research.

To understand the computational problem the visual system must solve in stereopsis, consider how images are generated when natural scenes are viewed binocularly. Vision depends on how light is reflected onto our two retinal surfaces from external surfaces in 3-D space. Surfaces can vary in reflectance, depth, and transmittance, all of which can affect the way that images are formed on the two retinae. One natural way to think about vision is as a reverse image formation process: given a series of images, what are their most likely causes? To answer this question in stereovision, the visual system must infer the transformations that relate the images in the two eyes, and recover the information that these transformations provide about the three-dimensional world.

Historically, the informational basis of stereopsis was thought to be specified by the pattern of binocular 'disparities' computed from the two images[1–11]. Disparity refers to the difference in the visual direction of two or more points in space. When two points are situated at different depths from the observer, their angular separation will be different in the two eyes. The difference between these separations is the disparity (or relative disparity) of image features, which can be used to infer their depth. In order to compute disparity, the visual system must have some means of solving the correspondence problem, that is, of determining which features in the two eyes correspond to a common surface feature in the 3-D world. Until recently, it was thought that the problem of stereovision was solved once correspondence was determined and disparity was computed. For this reason, virtually all of the theoretical work in stereovision over the past few decades has focused on developing solutions to the problem of matching.

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